November 30, 2022

Handout for Week 13:

Ongoing Projects: Dialogic Pragmatics and Monadologics

Philosophy of Language.
Metavocabularies of Reason:
Pragmatics, Semantics, and Logic
https://sites.pitt.edu/~rbrandom/Courses

Plan:

I. Dialogic Pragmatics
II. Monadologics

I. Dialogic Pragmatics

1. Concrete Models of Vocabularies

Here is *a* concrete model:

Lexicon: {0,1,2,3,4,5,6}

This vocabulary contains 193 reasons-for (implications), as follows:

'{0, 1, 2, 3, 5}|~6', '{0, 1, 2, 4, 5}|~6', '{0, 1, 2, 4}|~6', '{0, 1, 2, 5, 6}|~3', '{0, 1, 2, 5}|~4',
'{0, 1, 2, 6}|~4', '{0, 1, 2}|~4', '{0, 1, 3, 4}|~2', '{0, 1, 3, 5}|~2', '{0, 1, 3, 5}|~6',
'{0, 1, 3, 6}|~5', '{0, 1, 3}|~2', '{0, 1, 3}|~5', '{0, 1, 4}|~6', '{0, 1, 5, 6}|~4',
'{0, 1, 5}|~3', '{0, 1, 5}|~6', '{0, 1, 6}|~2', '{0, 1, 6}|~3', '{0, 1}|~3',
'{0, 1}|~4', '{0, 1}|~6', '{0, 2, 3, 4, 5}|~6', '{0, 2, 3, 4, 6}|~5', '{0, 2, 3, 4}|~5',
'{0, 2, 3, 4}|~6', '{0, 2, 3, 5}|~4', '{0, 2, 3, 6}|~1', '{0, 2, 3, 6}|~4', '{0, 2, 3, 6}|~5',
'{0, 2, 3}|~6', '{0, 2, 4, 5, 6}|~3', '{0, 2, 4, 5}|~1', '{0, 2, 4, 6}|~1', '{0, 2, 4, 6}|~5',
'{0, 2, 4}|~1', '{0, 2, 4}|~3', '{0, 2, 5, 6}|~3', '{0, 2, 5}|~1', '{0, 2, 5}|~4',
'{0, 2, 5}|~6', '{0, 2, 6}|~1', '{0, 2, 6}|~3', '{0, 2}|~4', '{0, 2}|~6',

'{0, 3, 4, 5}|~6', '{0, 3, 4, 6}|~5', '{0, 3, 4}|~2', '{0, 3, 4}|~5', '{0, 3, 4}|~6',
'{0, 3, 5, 6}|~2', '{0, 3, 5}|~1', '{0, 3, 5}|~2', '{0, 3, 5}|~6', '{0, 3, 6}|~1',
'{0, 3, 6}|~4', '{0, 3, 6}|~5', '{0, 3}|~5', '{0, 4, 5, 6}|~3', '{0, 4, 5}|~1',
'{0, 4, 5}|~2', '{0, 4, 5}|~3', '{0, 4, 6}|~1', '{0, 4, 6}|~2', '{0, 4, 6}|~3',
'{0, 4}|~2', '{0, 4}|~3', '{0, 4}|~6', '{0, 5, 6}|~1', '{0, 5, 6}|~2',
'{0, 5}|~4', '{0, 5}|~6', '{0, 6}|~1', '{0, 6}|~2', '{0, 6}|~5',
'{0}|~3', '{0}|~5', '{1, 2, 3, 4}|~0', '{1, 2, 3, 4}|~6', '{1, 2, 3, 5}|~0',
'{1, 2, 3, 6}|~0', '{1, 2, 3, 6}|~4', '{1, 2, 3}|~4', '{1, 2, 3}|~5', '{1, 2, 4, 5, 6}|~0',
'{1, 2, 4, 5}|~3', '{1, 2, 4, 5}|~6', '{1, 2, 4, 6}|~0', '{1, 2, 4, 6}|~3', '{1, 2, 4}|~3',
'{1, 2, 4}|~5', '{1, 2, 5, 6}|~4', '{1, 2, 5}|~0', '{1, 2, 5}|~6', '{1, 2}|~0',
'{1, 2}|~4', '{1, 2}|~5', '{1, 3, 4, 6}|~2', '{1, 3, 4, 6}|~5', '{1, 3, 4}|~0',
'{1, 3, 4}|~5', '{1, 3, 4}|~6', '{1, 3, 5, 6}|~0', '{1, 3, 5, 6}|~4', '{1, 3, 5}|~2',
'{1, 3, 5}|~6', '{1, 3, 6}|~0', '{1, 3, 6}|~2', '{1, 3, 6}|~4', '{1, 3, 6}|~5',
'{1, 3}|~2', '{1, 3}|~4', '{1, 3}|~5', '{1, 4, 5, 6}|~0', '{1, 4, 5, 6}|~3',
'{1, 4, 5}|~0', '{1, 4, 5}|~3', '{1, 4, 5}|~6', '{1, 4, 6}|~5', '{1, 4}|~2',
'{1, 5, 6}|~2', '{1, 5, 6}|~3', '{1, 6}|~0', '{1}|~0', '{1}|~2',
'{1}|~6', '{2, 3, 4, 6}|~0', '{2, 3, 4, 6}|~1', '{2, 3, 4}|~0', '{2, 3, 4}|~6',
'{2, 3, 5, 6}|~0', '{2, 3, 5}|~0', '{2, 3, 5}|~4', '{2, 3, 5}|~6', '{2, 3, 6}|~4',
'{2, 3}|~1', '{2, 4, 5, 6}|~0', '{2, 4, 5, 6}|~3', '{2, 4, 5}|~6', '{2, 4, 6}|~1',
'{2, 4, 6}|~5', '{2, 4}|~0', '{2, 5, 6}|~0', '{2, 5, 6}|~1', '{2, 5, 6}|~4',
'{2, 5}|~0', '{2, 5}|~1', '{2, 6}|~0', '{2, 6}|~1', '{2, 6}|~4',
'{2, 6}|~5', '{2}|~3', '{2}|~6', '{3, 4, 5, 6}|~0', '{3, 4, 5, 6}|~1',
'{3, 4, 5}|~0', '{3, 4, 5}|~1', '{3, 4, 5}|~6', '{3, 4, 6}|~0', '{3, 4, 6}|~2',
'{3, 4, 6}|~5', '{3, 4}|~0', '{3, 4}|~1', '{3, 4}|~2', '{3, 4}|~5',
'{3, 5, 6}|~0', '{3, 5, 6}|~1', '{3, 5, 6}|~4', '{3, 5}|~2', '{3, 5}|~4',
'{3, 5}|~6', '{3, 6}|~0', '{3, 6}|~1', '{3, 6}|~2', '{3, 6}|~5',
'{3}|~0', '{3}|~6', '{4, 5, 6}|~0', '{4, 5, 6}|~2', '{4, 5}|~2',
'{4, 5}|~3', '{4, 6}|~0', '{4, 6}|~2', '{4}|~0', '{4}|~2',
'{5, 6}|~3', '{5, 6}|~4', '{5}|~2', '{5}|~3', '{5}|~4',
'{5}|~6', '{6}|~0', '{6}|~2'
^^

This vocabulary contains 198 reasons-against (incompatibilities), as follows:

'{0, 1, 2, 3, 4}#5', '{0, 1, 2, 3, 4}#6', '{0, 1, 2, 3, 5, 6}#4', '{0, 1, 2, 3, 5}#4', '{0, 1, 2, 3, 6}#4',
'{0, 1, 2, 3}#5', '{0, 1, 2, 4, 5, 6}#3', '{0, 1, 2, 4, 5}#3', '{0, 1, 2, 4, 6}#3', '{0, 1, 2, 4}#5',
'{0, 1, 2, 5}#3', '{0, 1, 2, 5}#4', '{0, 1, 2, 5}#6', '{0, 1, 2, 6}#5', '{0, 1, 2}#6',

'{0, 1, 3, 4}#5', '{0, 1, 3, 4}#6', '{0, 1, 3, 5, 6}#4', '{0, 1, 3, 5}#2', '{0, 1, 3, 5}#4',
'{0, 1, 3, 5}#6', '{0, 1, 3, 6}#4', '{0, 1, 3, 6}#5', '{0, 1, 3}#4', '{0, 1, 3}#5',
'{0, 1, 4, 5, 6}#3', '{0, 1, 4, 5}#2', '{0, 1, 4, 5}#3', '{0, 1, 4, 5}#6', '{0, 1, 4, 6}#3',
'{0, 1, 4, 6}#5', '{0, 1, 4}#3', '{0, 1, 4}#6', '{0, 1, 5, 6}#2', '{0, 1, 5, 6}#3',
'{0, 1, 5, 6}#4', '{0, 1, 5}#3', '{0, 1, 6}#2', '{0, 1, 6}#3', '{0, 1, 6}#4',
'{0, 1}#5', '{0, 1}#6', '{0, 2, 3, 4, 5, 6}#1', '{0, 2, 3, 4, 5}#1', '{0, 2, 3, 4, 6}#1',
'{0, 2, 3, 4}#6', '{0, 2, 3, 5}#1', '{0, 2, 3, 5}#6', '{0, 2, 3, 6}#4', '{0, 2, 3, 6}#5',
'{0, 2, 3}#5', '{0, 2, 4, 5}#1', '{0, 2, 4, 6}#3', '{0, 2, 4}#5', '{0, 2, 4}#6',
'{0, 2, 5, 6}#1', '{0, 2, 5, 6}#3', '{0, 2, 5}#3', '{0, 2, 5}#4', '{0, 2, 5}#6',
'{0, 2, 6}#1', '{0, 2, 6}#4', '{0, 2, 6}#5', '{0, 2}#4', '{0, 3, 4, 5, 6}#1',
'{0, 3, 4, 5}#1', '{0, 3, 4, 6}#1', '{0, 3, 4, 6}#2', '{0, 3, 4}#1', '{0, 3, 5, 6}#1',
'{0, 3, 5, 6}#2', '{0, 3, 5}#1', '{0, 3, 6}#1', '{0, 3}#4', '{0, 3}#6',
'{0, 4, 5, 6}#1', '{0, 4, 5}#2', '{0, 4, 5}#6', '{0, 4, 6}#1', '{0, 4, 6}#2',
'{0, 4, 6}#5', '{0, 4}#2', '{0, 4}#3', '{0, 4}#5', '{0, 5, 6}#2',
'{0, 5, 6}#4', '{0, 5}#1', '{0, 5}#4', '{0, 5}#6', '{0, 6}#1',
'{0, 6}#3', '{0, 6}#5', '{0}#2', '{1, 2, 3, 4, 5}#0', '{1, 2, 3, 4, 5}#6',
'{1, 2, 3, 4, 6}#0', '{1, 2, 3, 4, 6}#5', '{1, 2, 3, 5, 6}#4', '{1, 2, 3, 5}#0', '{1, 2, 3}#6',
'{1, 2, 4, 5, 6}#3', '{1, 2, 4, 5}#0', '{1, 2, 4}#5', '{1, 2, 5, 6}#0', '{1, 2, 5}#4',
'{1, 2, 5}#6', '{1, 2, 6}#0', '{1, 2, 6}#3', '{1, 2, 6}#5', '{1, 2}#5',
'{1, 2}#6', '{1, 3, 4, 5, 6}#0', '{1, 3, 4, 5, 6}#2', '{1, 3, 4, 5}#0', '{1, 3, 4, 6}#0',
'{1, 3, 4}#0', '{1, 3, 5, 6}#0', '{1, 3, 5}#0', '{1, 3, 6}#0', '{1, 3, 6}#2',
'{1, 3}#4', '{1, 3}#6', '{1, 4, 5, 6}#0', '{1, 4, 5}#2', '{1, 4, 5}#6',
'{1, 4, 6}#0', '{1, 4, 6}#5', '{1, 4}#3', '{1, 4}#6', '{1, 5, 6}#2',
'{1, 5, 6}#4', '{1, 5}#0', '{1, 5}#2', '{1, 6}#0', '{1, 6}#2',
'{1, 6}#3', '{1, 6}#4', '{1}#2', '{1}#5', '{1}#6',
'{2, 3, 4, 5, 6}#1', '{2, 3, 4, 5}#6', '{2, 3, 4, 6}#0', '{2, 3, 4, 6}#5', '{2, 3, 5, 6}#0',
'{2, 3, 5, 6}#4', '{2, 3, 5}#0', '{2, 3, 6}#1', '{2, 4, 5, 6}#3', '{2, 4, 5}#0',
'{2, 4, 5}#1', '{2, 4, 6}#0', '{2, 4}#0', '{2, 4}#5', '{2, 4}#6',
'{2, 5, 6}#0', '{2, 5, 6}#1', '{2, 5}#1', '{2, 5}#4', '{2, 5}#6',
'{2, 6}#1', '{2, 6}#4', '{2, 6}#5', '{2}#0', '{2}#1',
'{2}#6', '{3, 4, 5, 6}#2', '{3, 4, 5}#6', '{3, 4, 6}#5', '{3, 4}#0',
'{3, 4}#1', '{3, 4}#6', '{3, 5, 6}#4', '{3, 6}#0', '{3, 6}#1',
'{3, 6}#4', '{3}#5', '{3}#6', '{4, 5, 6}#0', '{4, 5, 6}#1',
'{4, 5, 6}#3', '{4, 5}#0', '{4, 5}#2', '{4, 5}#6', '{4, 6}#1',
'{4, 6}#2', '{4, 6}#3', '{4, 6}#5', '{4}#6', '{5, 6}#0',
'{5, 6}#2', '{5, 6}#4', '{5}#1', '{5}#3', '{6}#1',
'{6}#2', '{6}#3', '{6}#4'

2. Summary of a dialogue conducted by Python program DP1 (Yao Fan):

Turn Number	Agent	Turn Target	Pragmatic Significance	Move	Claimant Acceptance Commitments	Claimant Rejection Commitments	Claimant Acceptance Entitlements	Claimant Rejection Entitlements	Critic Acceptance Commitments	Critic Rejection Commitments	Critic Acceptance Entitlements	Critic Rejection Entitlements
0	CL	None	Proposal	0,2,5 |~ 3	0,2,3,5	-	0,2,3,5	-	-	-	-	-
1	CR	0	Premise Challenge	0 # 5	0,2,3,5	-	0,2	-	0	5	0	5
2	CL	1	Conclusion Challenge	0,2,6 |~ 5	0,2,3,5,6	-	0,2,3,5,6	-	0	5	0	-
3	CR	0	Conclusion Challenge	0 # 3	0,2,3,5,6	-	0,2,5,6	-	0	3,5	0	3
4	CL	3	Conclusion Challenge	0,6 |~ 3	0,2,3,5,6	-	0,2,3,5,6	-	0	3,5	0	-
5	CR	4	Premise Challenge	0 # 6	0,2,3,5,6	-	0,2	-	0	3,5,6	0	3,5,6
6	CL	5	Conclusion Challenge	0,2 |~ 6	0,2,3,5,6	-	0,2,3,5,6	-	0	3,5,6	0	-
7	CR	2	Conclusion Challenge	0,1 # 5	0,2,3,5,6	-	0,2,3,6	-	0,1	3,5,6	0,1	5
8	CL	7	Conclusion Challenge	0 |~ 5	0,2,3,5,6	-	0,2,3,5,6	-	0,1	3,5,6	0,1	-
9	CR	8	Conclusion Challenge	1,2 # 5	0,2,3,5,6	-	0,2,3,6	-	0,1,2	3,5,6	0,1,2	5
10	CL	9	Premise Challenge	2 # 1	0,2,3,5,6	1	0,2,3,5,6	1	0,1,2	3,5,6	0,2	-
11	CR	10	Conclusion Challenge	0 |~ 1	0,2,3,5,6	1	0,2,3,6	-	0,1,2	3,5,6	0,1,2	5
12	CL	9	Conclusion Challenge	6 |~ 5	0,2,3,5,6	1	0,2,3,5,6	-	0,1,2	3,5,6	0,1,2	-
13	CR	12	Premise Challenge	2 # 6	0,2,3,5,6	1	0,2	-	0,1,2	3,5,6	0,1,2	3,5,6

Proposal is rejected. (Claimant CL cannot sustain entitlement to 3.)
Propositional Common ground, to which both interlocutors end up committed and entitled after their critical inquiry, is 0,2.

3. DP2—Building a Shared Vocabulary

Agent [random] is committed to accept the following sentences:
['a_0', 'a_4']
Agent [random] is committed to reject the following sentences:
['a_3', 'a_6']
Agent [random] is indifferent about the following sentences:
['a_1', 'a_2', 'a_5']

random	a_0 ✓	a_1	a_2	a_3 ✗	a_4 ✓	a_5	a_6 ✗
Prob_to_Support	0.37	0.07	0.07	0.01	0.37	0.07	0.01
Prob_to_Resist	0.01	0.07	0.07	0.37	0.01	0.07	0.37

Agent [cooperative] is committed to accept the following sentences:
['a_4', 'a_6']
Agent [cooperative] is committed to reject the following sentences:
['a_0', 'a_3']
Agent [cooperative] is indifferent about the following sentences:
['a_1', 'a_2', 'a_5']

cooperative	a_0 ✗	a_1	a_2	a_3 ✗	a_4 ✓	a_5	a_6 ✓
Prob_to_Support	0.01	0.07	0.07	0.01	0.37	0.07	0.37
Prob_to_Resist	0.37	0.07	0.07	0.37	0.01	0.07	0.01

Num	Proposer	Proposee	Proposal	Response	Newly Accepted
0	random	cooperative	[1, 2, 4, 5, 6]|~0	accept	[1, 2, 4, 5, 6]|~0
1	cooperative	random	[2, 3, 4, 6]#1	accept with conditions	[2, 3, 4, 6]#1, [2, 3, 4, 5, 6]|~1
2	random	cooperative	[0]|~4	accept	[0]|~4
3	cooperative	random	[0]|~6	reject	
4	random	cooperative	[0, 3, 6]|~5	accept	[0, 3, 6]|~5
5	cooperative	random	[5]|~1	reject	
6	random	cooperative	[1, 4, 5]|~0	accept	[1, 4, 5]|~0
7	cooperative	random	[0, 4]|~1	accept	[0, 4]|~1
8	random	cooperative	[1, 6]|~5	accept	[1, 6]|~5
9	cooperative	random	[0, 1, 5, 6]|~4	accept with conditions	[0, 1, 5, 6]|~4, [0, 1, 3, 5, 6]#4
10	random	cooperative	[4]|~0	accept with conditions	[4]|~0, [1, 3, 4, 5]#0, [1, 2, 3, 4, 5]|~0
11	cooperative	random	[0, 2, 5, 6]|~4	accept with conditions	[0, 2, 5, 6]|~4, [0, 2, 3, 5, 6]#4

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In this inquiry, 153 reasons-for were proposed and 124 were accepted, which makes the acceptance rate of reasons-for 0.81
In this inquiry, 78 reasons-against were proposed and 67 were accepted, which makes the acceptance rate of reasons-against 0.86
Overall, 231 proposals were made and 191 were accepted. The overall acceptance rate is thus 0.83

Example of conditional agreements in DP2:

```
Tree<'Round 1'>
└── cooperative: [2, 3, 4, 6]#1
    └── random: [2, 3, 4, 6]#1, [2, 3, 4, 5, 6]|~1
        └── cooperative: ✓

Tree<'Round 80'>
└── random: [4, 6]|~0
    ├── cooperative: [4, 6]|~0, [2, 4, 6]#0
    │   └── random: ✕
    ├── cooperative: [4, 6]|~0, [1, 3, 4, 5, 6]#0
    │   └── random: ✕
    └── cooperative: [4, 6]|~0, [2, 3, 4, 6]#0
        └── random: [4, 6]|~0, [2, 3, 4, 6]#0, [2, 3, 4, 5, 6]|~0
            └── cooperative: ✓

Tree<'Round 147'>
└── cooperative: [1, 2, 5]|~6
    └── random: [1, 2, 5]|~6, [0, 1, 2, 3, 4, 5]#6
        └── cooperative: ✓
```

II. Monadologics

1. Two views of the intensionality of our conditional and negation.

   i) To determine whether or not Γ |~ A→B or Γ |~ ¬A, must look outside the consequences of Γ, to the consequences of Γ,A.

In particular, Γ |~ A→B iff Γ,A |~ B and Γ |~ ¬A iff Γ,A |~ ⊥.

   ii) The logically complex consequences of Γ accordingly encode information about the consequences of other premise-sets.

2. The idea of monadologics.

   a) Observation: Our conditional and negation are exclusively *upward*-looking. They codify *all* of the reason relations *only* of Γs supersets: its *upward cone* in the lattice of subsets of the language.
   In *this* sense, our conditional and negation (with their aggregative helper-monkeys of conjunction and disjunction) are an expressively *in*complete *set* of logical connectives.
   (This is not the sense in which Dan showed NM-MS to be expressively complete relative to base vocabularies.)
   For these expressive logical connectives only codify in the consequences of *one* premise-set the reason relations in which *some*, but *not all* other premise-sets stand.

   b) This observation invites a further expressive ambition:

   An *expressively complete **set*** of logical connectives would do for *every* possible premise-set defined on the lexicon of the material vocabulary in question what *our* connectives do for the *supersets* of a given premise-set: codify the reason relations of *all* premise-sets in the logically complex consequences of *each* premise-set.

   A set of sentential connectives that does that is what I call a "monadologic"—after Leibniz's monads.

A ***monadologic*** would be a set of connectives (defined by rules in a sequent calculus) such that the logically complex consequences of *each and every* premise-set codify the implications and incompatibilities-incoherences of *every other* premise-set, in the same sense in which the conditional and negation of NM-MS do for the *supersets* of the premise-set whose logically complex consequences they articulate.

Can there be such a thing? Is the conception even, in the end, coherent?

I don't know.

3. A possible means for defining a monadologic: A *downward* conditional.

The “upward-looking” character of our conditional is codified in the Deduction-Detachment (DD) (Dual Ramsey) criterion of adequacy on the right rule for the conditional:

Deduction-Detachment (DD) Condition on Conditionals: $\Gamma|\!\sim A\rightarrow B$ iff $\Gamma,A|\!\sim B$.

This looks at the consequences of *adding* A to Γ.

We want a *downward conditional* satisfying something like : $\Gamma|\!\sim A\rightarrow^{-}B$ iff $\Gamma\text{-}A|\!\sim B$.

It would look at the consequence of *subtracting* A from Γ.

The idea is that these “downward” connectives would allow us to capture, in the implications by Γ of logically complex sentences, the implications and incoherences of *subsets* of Γ, in a way parallel to how the “upward” connectives of NM-MS codify the implications and incoherences of *supersets* of Γ in its implications of logically complex sentences.

But functioning downward conditionals would be able to express much more than the reason relations only of subsets of Γ. For a premise-set Γ can be connected to another premise-set Δ that merely overlaps, or even is disjoint from Γ, by a sequence of *additions* of premises in Δ and *subtractions* of premises in Γ but not in Δ.

4. A concrete proposal.

Here is NM-MS:

$$\text{L}\rightarrow:\quad \frac{\Gamma|\!\sim\Theta,A \qquad B,\Gamma|\!\sim\Theta}{A\rightarrow B,\Gamma|\!\sim\Theta} \qquad\qquad \text{R}\rightarrow:\quad \frac{A,\Gamma|\!\sim\Theta,B}{\Gamma|\!\sim\Theta,A\rightarrow B}$$

$$\text{L\&}:\quad \frac{\Gamma,A,B|\!\sim\Theta}{\Gamma,A\&B|\!\sim\Theta} \qquad\qquad \text{R\&}:\quad \frac{\Gamma|\!\sim A,\Theta \qquad \Gamma|\!\sim B,\Theta}{\Gamma|\!\sim A\&B,\Theta}$$

$$\text{L}\vee:\quad \frac{A,\Gamma|\!\sim\Theta \qquad B,\Gamma|\!\sim\Theta}{A\vee B,\Gamma|\!\sim\Theta} \qquad\qquad \text{R}\vee:\quad \frac{\Gamma|\!\sim A,B,\Theta}{\Gamma|\!\sim A\vee B,\Theta}$$

$$\text{L}\neg:\quad \frac{\Gamma|\!\sim A,\Theta}{\neg A,\Gamma|\!\sim\Theta} \qquad\qquad \text{R}\neg:\quad \frac{A,\Gamma|\!\sim\Theta}{\Gamma|\!\sim\neg A,\Theta}$$

Try: Add the downward conditional $\rightarrow^{-}$ with the two new rules:

$$\mathbf{R\rightarrow^{-}}:\quad \frac{\boldsymbol{\Gamma\text{-}A|\!\sim\Theta,B}}{\boldsymbol{\Gamma|\!\sim\Theta,A\rightarrow^{-}B.}}$$

$$\mathbf{L\rightarrow^{-}}:\quad \frac{\boldsymbol{B,\Gamma\text{-}A|\!\sim\Theta}}{\boldsymbol{A\rightarrow^{-}B,\Gamma\text{-}A|\!\sim\Theta.}}$$

5. Desiderata:

i) consistency, ii) conservativeness, iii) detachment from downward conditional, iv) path-independence of mixed upward and downward conditionals, v) preservation of CO.

6. Hopeful results:

a) CO Preservation (v):

$B,\Gamma\text{-}A \mid\sim B, \Pi$	CO
$A\rightarrow^{-}B, \Gamma\text{-}A \mid\sim B,\Pi$	$L\rightarrow^{-}$
$A\rightarrow^{-}B, \Gamma \mid\sim A\rightarrow^{-}B, \Pi$	$R\rightarrow^{-}$

This proves CO for $\rightarrow^{-}$.

And it holds whether or not $A\in\Gamma$.

b) Detachment (iii):

The middle line of the proof above is detachment for downward conditionals: the downward conditional, together with a premise-set from which its antecedent has been removed, implies the consequent of the downward conditional.

So only CO in the base vocabulary (assumed in the first line) is required for these two results.

c) Path-independence of downward conditionals:

For $\Delta = \Gamma + A\ \text{-}B$, show that

$\Gamma \mid\sim A\rightarrow(B\rightarrow^{-}C)$ iff $\Gamma \mid\sim B\rightarrow^{-}(A\rightarrow C)$.

Left to right:

Given $\Gamma \mid\sim A\rightarrow(B\rightarrow^{-}C)$.

By $R\rightarrow$, this holds iff $\Gamma,A \mid\sim B\rightarrow^{-}C$.

r) By $R\rightarrow^{-}$, this holds iff $\{\Gamma,A\}\text{-}B \mid\sim C$.

We are trying to show that $\Gamma \mid\sim B\rightarrow^{-}(A\rightarrow C)$.

By $R\rightarrow^{-}$, this holds iff $\Gamma\text{-}B \mid\sim A\rightarrow C$.

r') By $R\rightarrow$, this holds iff $\Gamma\text{-}B,A \mid\sim C$.

(r) and (r') are equivalent.
This will work to show the other direction, too.

d) Path-independence of mixed upward and downward conditionals (iv):

Still need to show that the order of upward and downward conditionals does not matter when we mix them.
Want to show that i) Γ |~ A→(B→$^-$C) iff ii) Γ |~ B→$^-$(A→C):

i) Γ |~ A→(B→$^-$C) iff Γ,A |~ B→$^-$C, by R→.
Γ,A |~ B→$^-$C iff iii) **Γ-B,A |~ C**, by R→$^-$.

ii) Γ |~ B→$^-$(A→C) iff Γ-B |~ A→C, by R→$^-$
Γ-B |~ A→C iff iii) **Γ-B, A |~ C**, by R→.

So that is as it should be.

e) Monadologicality:

I conclude that the logically complex consequences of every Γ encode the consequences of every Δ.
Suppose Δ-($\Delta\cap\Gamma$) = {$X_1 \ldots X_n$}, the set of sentences in Δ but not Γ, and
Γ-($\Delta\cap\Gamma$) = {$Y_1 \ldots Y_m$}, the set of sentences in Γ but not in Δ.
Then Δ |~ A iff Γ |~ Y_1→(Y_2→(Y_3→…Y_m)…)) → (X_1→$^-$(X_2→$^-$(X_3→$^-$…X_n)…)) →$^-$A).

So, great.
It looks as though NM-MS + R→$^-$ and L→$^-$ (downward conditional rules) is a monadologic.

7. The worm in the apple:

Not so fast.

If, like the rules of NM-MS, R→$^-$ in particular, is *reversible*, then the effect of adding the downward conditional is *not conservative* over the base vocabulary—even though all the rules are simplifying rules, which normally guarantees conservativeness.

Ulf Hlobil offered the following argument (summarizing concerns Shuhei Shimamura and Dan Kaplan wrestled with the last time we made a serious run at monadologicality, more than 5 years ago):

1. Suppose Γ |~ C, for Γ containing neither A nor B.
2. Then Γ,B |~ B→$^-$C, by R→$^-$, since by hypothesis Γ-B = Γ and Γ|~C.
3. So Γ,A,B -A |~ B→$^-$C, since A∉Γ, so Γ,A,B -A = Γ,B, and by (2) Γ,B |~ B→$^-$C.
4. So Γ,A,B |~ A→$^-$(B→$^-$C), by R→$^-$.
5. So Γ,A&B |~ A→$^-$(B→$^-$C), by L&.
6. So Γ,A&B, -A |~ B→$^-$C, by reversing R→$^-$.
7. Γ,A&B, -A -B |~ C, by reversing R→$^-$.
8. So Γ,A&B |~ C, since A∉Γ,A&B and B∉Γ,A&B.
9. So Γ,A,B |~C, by reversing L&.

In this way, we have derived Γ,A&B |~ C from Γ|~C, and indeed, Γ,A,B |~ C, for arbitrary A,B.

That need not have held before introducing →$^-$.

So *if* R→$^-$ is reversible, its addition to NM-MS is not conservative (given reversibility of L&).

Open Questions:

Can we get a useful monadologicality result if we do *not* treat R→$^-$ as a *reversible* rule?

Are there other such difficulties?

Note that I have not shown the system even to be consistent.

So I do not at this point know whether anything like this idea can be made to work.